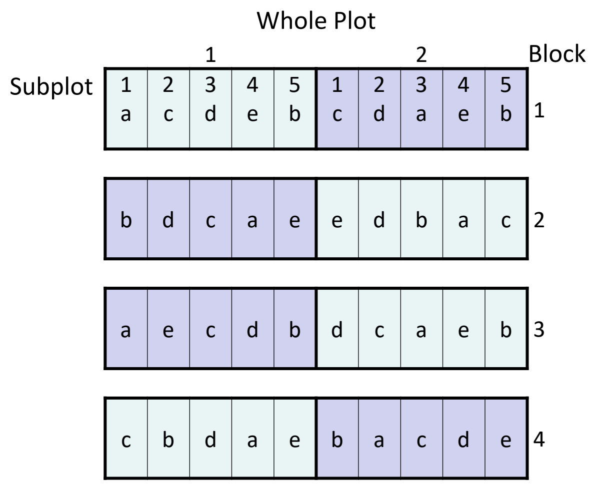
Split Plot Designs

Field Layout



Whole plot treatment: 1 2   Subplot treatment: a b c d e

Split-Plot Designs

Advantages

Spatial

- Different treatment sizes; e.g. field equipment, irrigation, etc.
- Minimize border effects; species, cultivars, fertility, etc.
- Reduce the physical size of an experiment

Temporal

- Repeated sampling over time

Other

- Expand inference; e.g. include varieties in a fertility trial
- Concentrate precision on sub-plot factor

Split-Plot Designs
Characteristics

- More than one plot size; i.e. plot size varies with treatment
- More than one error term; at least one per plot size
- More than one standard error for the mean; one for each treatment and additional ones for comparisons of treatment combinations

Split-Plot Designs
Linear Additive Model

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + W_j + BW_{ij} + \omega_{(ij)} + S_k + BS_{ik} + WS_{jk} + BWS_{ijk}$$

Where:

Y_{ijk} =	variable to be analyzed from the k^{th} experimental unit
μ =	overall mean
B_i =	effect of the i^{th} block
$\delta_{(i)}$ =	restriction error associated with blocks
W_j =	effect of the j^{th} whole-plot treatment
BW_{ij} =	whole-plot error (a), NID(0,s^2)
$\omega_{(ij)}$ =	restriction error associated with whole plots
S_k =	effect of the k^{th} sub-plot treatment
BS_{ik} =	interaction effect of the effect of the i^{th} level of B with effect of the k^{th} level of S (generally pooled into error b)
WS_{jk} =	interaction effect of the effect of the j^{th} level of W with effect of the k^{th} level of S
BWS_{ijk} =	sub-plot error (b), NID(0,s^2)

Split-Plot Design

Expected Mean Squares

Source	b R i	w F j	s F k	EMS
B_i	1	w	s	$\sigma^2 + s\sigma^2_\omega + ws\sigma^2_\delta + ws\sigma^2_B$
$\delta_{(i)}$	1	w	s	$\sigma^2 + s\sigma^2_\omega + ws\sigma^2_\delta$
W_j	b	0	s	$\sigma^2 + s\sigma^2_\omega + s\sigma^2_{BW} + bs\Phi(W)$
BW_{ij}	1	0	s	$\sigma^2 + s\sigma^2_\omega + s\sigma^2_{BW}$
$\omega_{(ij)}$	1	1	s	$\sigma^2 + s\sigma^2_\omega$
S_k	b	w	0	$\sigma^2 + w\sigma^2_{BS} + bw\Phi(S)$
BS_{ik}	1	w	0	$\sigma^2 + w\sigma^2_{BS}$
WS_{jk}	b	0	0	$\sigma^2 + \sigma^2_{BWS} + b\Phi(WS)$
BWS_{ijk}	1	0	0	$\sigma^2 + \sigma^2_{BWS}$

Split-Plot Design

Herbicide Trial Example

Factors:

Block	4
Variety (whole plot)	3
Herbicide (sub plot)	5

Herbicide Trial Example

Field Layout

Variety			1					3					2		
Herbicide	5	2	3	1	4	3	5	4	2	1	1	4	3	2	5
Variety			2					3					1		
Herbicide	4	1	3	5	2	3	1	5	4	2	2	1	3	5	4
Variety			2					1					3		
Herbicide	3	1	4	5	2	3	4	2	1	5	4	5	2	1	3
Variety			3					2					1		
Herbicide	4	5	3	1	2	1	2	3	4	5	4	2	3	1	5

Herbicide Trial Example

Expected Mean Squares

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + V_j + BV_{ij} + \omega_{(ij)} + H_k + BH_{ik} + VH_{jk} + BVH_{ijk}$$

Source	b R i	v F j	h F k	EMS
B_i	1	v	h	
$\delta_{(i)}$	1	v	h	
V_j	b	0	h	
BV_{ij}	1	0	h	
$\omega_{(ij)}$	1	1	h	
H_k	b	v	0	
BH_{ik}	1	v	0	
VH_{jk}	b	0	0	
BVH_{ijk}	1	0	0	

Herbicide Trial Example

Expected Mean Squares

$$Y_{ijk} = \mu + B_i + \delta_{(i)} + V_j + BV_{ij} + \omega_{(ij)} + H_k + BH_{ik} + VH_{jk} + BVH_{ijk}$$

	Source	b R i	v F j	h F k	EMS
	B_i	1	v	h	$\sigma^2 + h\sigma^2_{\omega} + vh\sigma^2_{\delta} + vh\sigma^2_{B}$
	$\delta_{(i)}$	1	v	h	$\sigma^2 + h\sigma^2_{\omega} + vh\sigma^2_{\delta}$
	V_j	b	0	h	$\sigma^2 + h\sigma^2_{\omega} + h\sigma^2_{BV} + bh\Phi(V)$
Error a	BV_{ij}	1	0	h	$\sigma^2 + h\sigma^2_{\omega} + h\sigma^2_{BV}$
	$\omega_{(ij)}$	1	1	h	$\sigma^2 + h\sigma^2_{\omega}$
	H_k	b	v	0	$\sigma^2 + v\sigma^2_{BH} + bv\Phi(H)$
Pool →	BH_{ik}	1	v	0	$\sigma^2 + v\sigma^2_{BH}$
	VH_{jk}	b	0	0	$\sigma^2 + \sigma^2_{BVH} + b\Phi(VH)$
Error b →	BVH_{ijk}	1	0	0	$\sigma^2 + \sigma^2_{BVH}$

Split-Plot Design

Standard Errors

<u>Main Effects Means</u>

SE for whole-plot means:

$$SED = \sqrt{\frac{2MS_a}{bs}} \qquad a_i - a_j$$

SE for subplot means:

$$SED = \sqrt{\frac{2MS_b}{bw}} \qquad b_i - b_j$$

Split-Plot Design

Standard Errors

Interaction Means

SED for subplot treatment means at the same whole-plot treatment level:

$$SED = \sqrt{\frac{2MS_b}{b}}$$

a_ib_j - a_ib_k

SED for whole-plot treatment means at the same subplot treatment level:

$$SED = \sqrt{\frac{2[(s-1)MS_b + MS_a]}{bs}}$$

a_ib_j - a_kb_j

SED for whole-plot treatment means at different subplot treatment levels:

$$SED = \sqrt{\frac{2[(s-1)MS_b + MS_a]}{bs}}$$

a_ib_j - a_kb_l

Split-Plot Design

Example – Alfalfa Establishment

Treatments:

Blocks (B) 4

Whole plots - Seed distribution method (D) 3

- 5.5-in drill
- 70-ft floater
- 30-ft broadcast

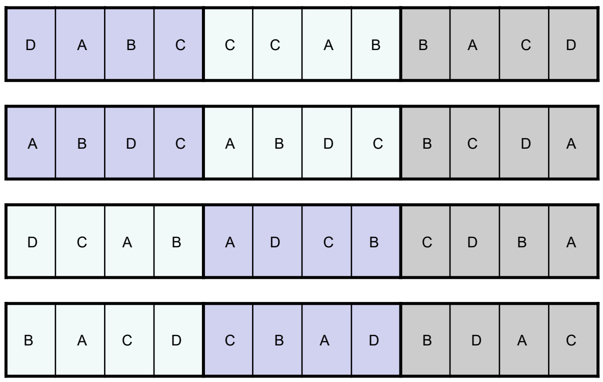
Subplots - Seed covering method (C) 4

- Cultipack before/after seeding
- Cultipack after seeding
- Harrow after seeding
- Control

Data source: Ted Bailey, PROC MIXED Workshop, Annual Meetings of the American Society of Agronomy, 17 Oct. 1998

Split-Plot Design

Example – Alfalfa Establishment



Whole plots:
- Drill
- Floater
- Broadcast

Subplots:
- A Cultipack b/a
- B Cultipack a
- C Harrow
- D Control

Split-Plot Design

Example – Alfalfa Establishment

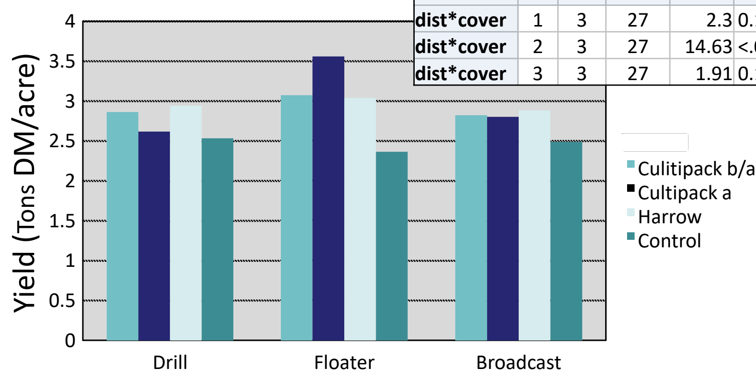
ANOVA

Source	DF	SS	Mean Square	F Value	Pr > F
blk	3	2.444217	0.814739		
dist	2	0.753404	0.376702	1.82	0.2411
Error a	6	1.241696	0.206949		
cover	3	2.221267	0.740422	11.28	<.0001
dist*cover	6	1.490796	0.248466	3.78	0.0073
Error b	27	1.772988	0.065666		

Split-Plot Design

Example – Alfalfa Establishment

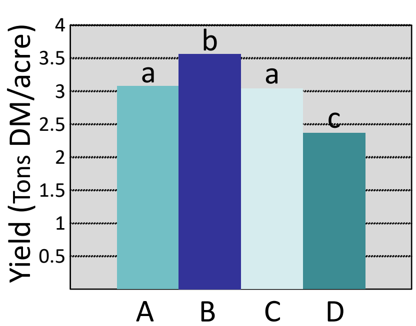
Tests of Effect Slices					
Effect	dist	Num DF	Den DF	F Value	Pr > F
dist*cover	1	3	27	2.3	0.1003
dist*cover	2	3	27	14.63	<.0001
dist*cover	3	3	27	1.91	0.1513



Split-Plot Design

Example – Alfalfa Establishment

Comparison of Covering Method for Floater Treatment



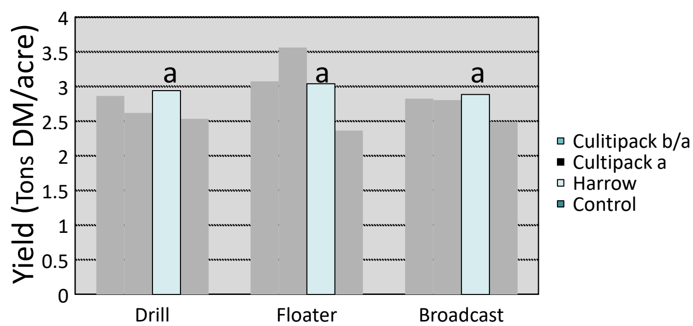
$$S_{\bar{d}} = \sqrt{\frac{2MS_b}{b}} = \sqrt{\frac{2(0.0657)}{4}} = 0.181$$

Split-Plot Design

Example – Alfalfa Establishment

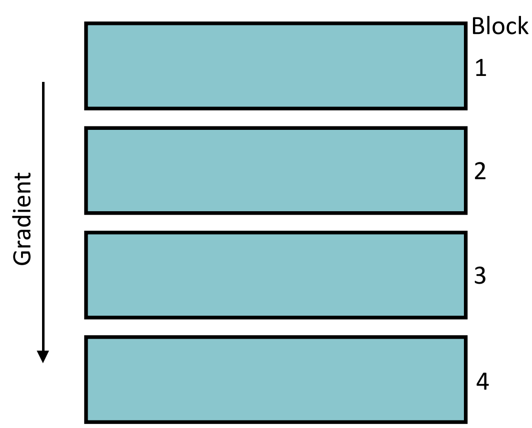
Comparison of Planting Method for Harrow Treatment

$$S_{\bar{d}} = \sqrt{\frac{2[(s-1)MS_b + MS_a]}{bs}} = \sqrt{\frac{2[(4-1)0.0657 + 0.2070]}{4(4)}} = 0.2247$$



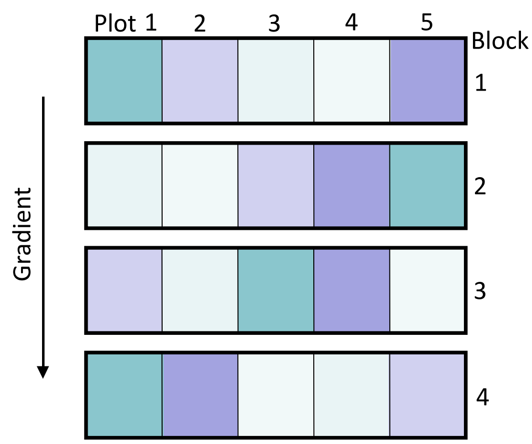
Split-Block Design

Field Layout



Split-Block Design

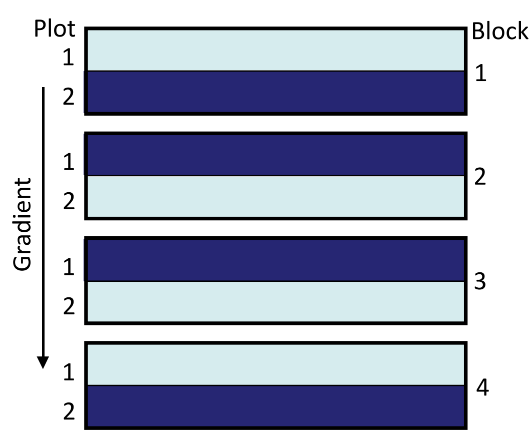
Field Layout



Treatment A plots = 1/5 block size

Split-Block Design

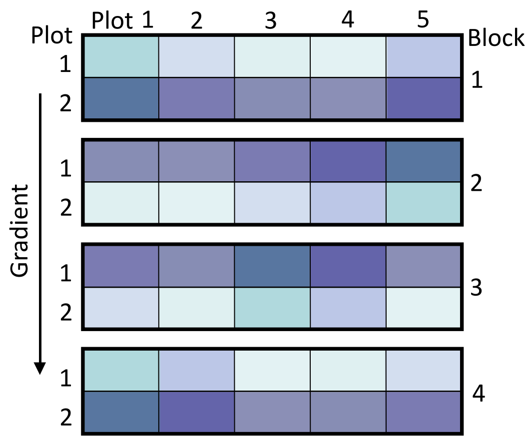
Field Layout



Treatment B plots = 1/2 block size

Split-Block Design

Field Layout



Treatment AB plots = 1/10 block size

Split-Block Design

Linear Additive Model

$$Y_{ijk} = \mu + R_i + \delta_{(i)} + A_j + RA_{ij} + \omega_{(ij)} + B_k + RB_{ik} + \lambda_{(ik)} + AB_{jk} + RAB_{ijk}$$

Where:

Y_{ijk} =	variable to be analyzed from the k^{th} experimental unit
μ =	overall mean
R_i =	effect of the i^{th} block
A_j =	effect of the j^{th} A treatment
RA_{ij} =	error a, NID$(0,\sigma^2)$
B_k =	effect of the k^{th} B treatment
RB_{ik} =	error b, NID$(0,\sigma^2)$
AB_{jk} =	interaction effect of the effect of the j^{th} level of A with effect of the k^{th} level of B
RAB_{ijk} =	error c, NID$(0,\sigma^2)$

Split-Block Design

Expected Mean Squares

	Source	r R i	a F j	b F k	EMS
	R_i	1	a	b	$\sigma^2 + a\sigma^2_\lambda + b\sigma^2_\omega + ab\sigma^2_\delta + ab\sigma^2_R$
	$\delta_{(i)}$	1	a	b	$\sigma^2 + a\sigma^2_\lambda + b\sigma^2_\omega + ab\sigma^2_\delta$
	A_j	r	0	b	$\sigma^2 + a\sigma^2_\lambda + b\sigma^2_{RA} + rb\Phi(A)$
Error a	RA_{ij}	1	0	b	$\sigma^2 + a\sigma^2_\lambda + b\sigma^2_{RA}$
	$\omega_{(ij)}$	1	1	b	$\sigma^2 + b\sigma^2_\omega$
	B_k	r	a	0	$\sigma^2 + a\sigma^2_\lambda + a\sigma^2_{RB} + ra\Phi(B)$
Error b	RB_{ik}	1	a	0	$\sigma^2 + a\sigma^2_\lambda + a\sigma^2_{RB}$
	$\lambda_{(ik)}$	1	a	1	$\sigma^2 + a\sigma^2_\lambda$
	AB_{jk}	r	0	0	$\sigma^2 + \sigma^2_{RAB} + r\Phi(AB)$
Error c	RAB_{ijk}	1	0	0	$\sigma^2 + \sigma^2_{RAB}$

Split-Block Design

Standard Errors

Main Effects Means

SED for A mean comparisons:

$$S_{\bar{d}} = \sqrt{\frac{2MS_a}{rb}} \qquad a_i - a_j$$

SED for B mean comparisons:

$$S_{\bar{d}} = \sqrt{\frac{2MS_b}{ra}} \qquad b_i - b_j$$

Split-Block Design

Standard Errors

Interaction Means

SED for A means at the same B treatment level:

$$S_{\bar{d}} = \sqrt{\frac{2[(b-1)MS_c + MS_a]}{rb}}$$

a_ib_j - a_kb_j

SED for B means at the same A treatment level:

$$S_{\bar{d}} = \sqrt{\frac{2[(a-1)MS_c + MS_b]}{ra}}$$

a_ib_j - a_ib_k

SED for A means at different B treatment levels:

$$S_{\bar{d}} = \sqrt{\frac{2[(ab-a-b)MS_c + aMS_a + bMS_b]}{rab}}$$

a_ib_j - a_kb_l